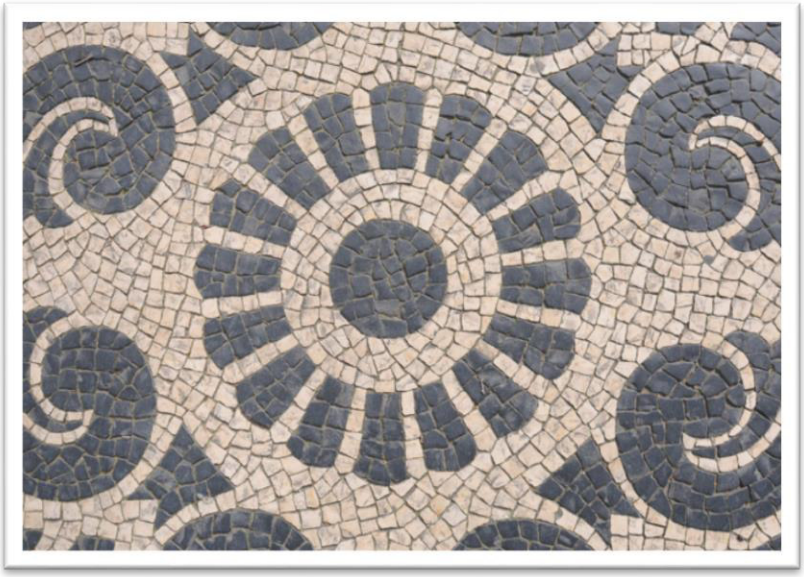
WOTCA 2017

Workshop on Operator Theory, Complex Analysis,
and Applications

BOOK OF ABSTRACTS



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Maximum entropy principle and Landau free energy inequality

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Abstract

In this talk, the following problem is considered: given two Hermitian matrices H and K and two real numbers x and y, determine a positive semidefinite matrix ρ such that the von Neumann entropy - $\mathrm{Tr}\rho\log\rho$ is maximum, subject to the condition that $\mathrm{Tr}\rho H = x$ and $\mathrm{Tr}\rho K = y$. This question arises in information theory and in statistical mechanics in connection with the maximum - entropy inference principle. To answer it, we use this principle and numerical range methods.

Classes of operators on some spaces of analytic functions

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Abstract

In this talk we use a characterization of the surjective isometries of some spaces of analytic functions to derive information on the hermitian operators and generalized bi-circular projections supported by the same spaces. The results presented are for the Zygmund space and for vector valued Bloch spaces.

Invariant subspaces of $H^2(\mathbb{T}^2)$ and $L^2(\mathbb{T}^2)$ preserving compatibility

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Abstract

It is well known that the operator of multiplication T_z on the Hardy space $H^2(\mathbb{T})$ is the model of a unilateral shift of multiplicity one, has no nontrivial reducing subspaces and, by the Beurling result, purely invariant subspaces are of the form $\phi H^2(\mathbb{T})$ where ϕ is an inner function. Similarly, the operator of multiplication L_z on $L^2(\mathbb{T})$ is a model of a bilateral shift of multiplicity one, by Helson its reducing subspaces are of the form $\chi_\delta L^2(\mathbb{T})$ where χ_δ is the characteristic function of a Borel set $\delta \subset \mathbb{T}$, while purely invariant subspaces are of the form $\psi H^2(\mathbb{T})$ where ψ is a unimodular function.

The similar description for $T_w, T_z \in H^2(\mathbb{T}^2)$ and $L_w, L_z \in L^2(\mathbb{T}^2)$ provides only a partial characterization. Recall, that two operators doubly commute if they commute and each of them commute with the adjoint of the other operator. The operators T_w, T_z are the model of doubly commuting unilateral shifts, have no nontrivial reducing subspaces and by Mandrekar inner functions describe only invariant subspaces where T_w, T_z preserve doubly commutativity. By Mandrekar and Ghatage reducing subspaces of L_w, L_z are of the form $\chi_\Delta L^2(\mathbb{T}^2)$ where $\Delta \subset \mathbb{T}^2$ and unimodular functions describe only invariant subspaces where L_w, L_z preserve doubly commutativity.

Our contribution is a characterization of invariant subspaces of $H^2(\mathbb{T}^2)$ and $L^2(\mathbb{T}^2)$ where T_w, T_z respectively L_w, L_z preserve compatibility. Recall that two isometries V_1, V_2 are compatible if projections on $\mathcal{R}(V_1^m)$ and $\mathcal{R}(V_2^n)$ commute for any nonnegative m, n. The class contains doubly commuting pairs, but is significantly bigger. Consequently, there is described a wider class of invariant subspaces. In particular it follows a class of invariant subspaces where restrictions are unilateral shifts and their unitary extensions, which obviously are bilateral shifts, are a proper part of the pair L_w, L_z.

An operator approach to diffraction from polygonal-conical screens

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Abstract

The main aim of the talk is to present how to construct resolvent operators for certain classes of boundary value problems in diffraction theory. We will use an operator theory approach to analyse problems of wave diffraction from polygonal-conical screens. These are formulated as boundary value problems for the three-dimensional Helmholtz equation with Dirichlet or Neumann conditions on a plane screen of polygonal-conical form (including unbounded and multiply-connected screens), in weak formulation. The method is based upon operator theoretical techniques in Hilbert spaces, such as the construction of matricial coupling relations and certain orthogonal projections. Various cross connections will be exposed, particularly considering classical Wiener-Hopf operators in Sobolev spaces as general Wiener-Hopf operators in Hilbert spaces.

This is based on joint work with R. Duduchava and F.-O. Speck.

Asymptotic behaviour of powers of composition operators

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Abstract

We study the asymptotic behaviour of the powers T^n of a continuous composition operator T on an arbitrary Banach space X of holomorphic functions on the open unit disc of the complex plane. We show that for composition operators, one has the following dichotomy: either the powers converge uniformly or they do not converge even strongly. We also show that uniform convergence of the powers of an operator $T \in L(X)$ is very much

related to the behaviour of the poles of the resolvent of T on the unit circle and that all poles of the resolvent of the composition operator T on X are algebraically simple. Our results are applied to study the asymptotic behaviour of semigroups of composition operators associated with holomorphic semiflows.

Singular Value Decompositions of products of projections

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Abstract

We characterize operators $T = PQ$, where P, Q are orthogonal projections in a Hilbert space and T admits a singular value decomposition. This is equivalent to the fact that $A = P - Q$ is diagonalizable. Many examples are presented, among Toeplitz, Hankel and Wiener-Hopf operators. There is also a differential geometry characterizations: if $T = PQ$ admits a singular value decomposition, then the generic parts of P and Q are joined by a minimal geodesic in the Grassmannian manifold with diagonalizable direction vector.

Joint work with E. Andruchow.

When is the inner factor of f-f(a) an interpolating Blaschke product for all a?

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Abstract

It is well known that functions that are bounded and analytic in the open unit disk, or analytic functions in other Hardy spaces, have inner-outer

factorizations. The possible inner factors are Blaschke products, singular inner functions, and products of these. An 'interpolating Blaschke product' is a special kind of infinite Blaschke product whose zeros satisfy a condition guaranteeing they are far apart.

In this talk, we will begin with the definitions of inner functions and interpolating sequences and describe some of their properties that motivate the question in the title. It has been known for some time that the atomic singular inner function f(z)= exp((z+1)/(z-1)) has the property of the title, and this motivated the question. A characterization of the functions satisfying this condition will be given along with a sketch of the main ideas of the proof. Finally, an application to descriptions of commutants of analytic Toeplitz operators on the Hardy Hilbert space will be given.

This talk is based on joint work of Eva Gallardo, Pam Gorkin, and the speaker.

Toral and Spherical Aluthge Transforms

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Abstract

We introduce two natural notions of multivariable Aluthge transforms (toral and spherical), and study their basic properties. In the case of 2-variable weighted shifts, we first prove that the toral Aluthge transform does not preserve (joint) hyponormality, in sharp contrast with the 1-variable case. Second, we identify a large class of 2-variable weighted shifts for which hyponormality is preserved under both transforms. Third, we consider whether these Aluthge transforms are norm-continuous. Fourth, we study how the Taylor and Taylor essential spectra of 2-variable weighted shifts behave under the toral and spherical Aluthge transforms; as a special case, we consider the Aluthge transforms of the Drury-Arveson 2-shift. Finally, we discuss the class of spherically quasinormal 2-variable weighted shifts, which are the fixed points for the spherical Aluthge transform.

The talk is based on joint work with Jasang Yoon.

The minus order and range additivity

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Abstract

In the late 1970s Nambooripad and Hartwig independently defined a partial order on regular elements of a semigroup with an idea (among others) of extending the order for idempotent elements: $e \leq f \Leftrightarrow e = ef = fe$. This partial order, known today as the minus order, extends in a natural fashion to the space of real or complex rectangular matrices, but difficulties arise in the attempt of extending it to bounded operators between arbitrary Hilbert spaces. In a paper from 2010 Šemrl gives an interesting discussion regarding these difficulties, defining the minus order between bounded operators A and B (equivalently) as $A \leq B \Leftrightarrow \overline{\mathrm{Im}(B)} = \overline{\mathrm{Im}(A)} \dotplus \overline{\mathrm{Im}(B-A)}$ and $\overline{\mathrm{Im}(B^*)} = \overline{\mathrm{Im}(A^*)} \dotplus \overline{\mathrm{Im}(B^*-A^*)}$ (interestingly, the same definition was independently given in a paper by Antezana, Corach and Stojanoff from 2006). We will shed an additional light to this discussion and in fact see that the minus order can be defined in terms of usual range additivity, or with (unbounded) generalized inverses for arbitrary Hilbert space operators. We will then highlight a few occasions from the literature when the minus order appeared implicitly in the study, and demonstrate its relation with the problems of (generalized) inverting the sum of two operators, systems of operator equations and optimization problems.

Mixed boundary value problems for the Laplace-Beltrami equation

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Abstract

Let $\mathcal{C}$ be a smooth hypersurface in $\mathbb{R}^3$ with a smooth boundary decomposed into two connected $\partial\mathcal{C} = \Gamma = \Gamma_D \cup \Gamma_N$ and non-intersecting $\Gamma_D \cap \Gamma_N = \emptyset$ parts. Let $\nu(\omega) = (\nu_1(\omega), \nu_2(\omega), \nu_3(\omega))$, $\omega \in \overline{\mathcal{C}}$ be the unit normal vector field on the surface $\mathcal{C}$. Let us consider the Laplace-Beltrami operator written in terms of the Günter's tangent derivatives $\Delta_{\mathcal{C}} := \mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2$, $\mathcal{D}_j := \partial_j - \nu_j \partial_\nu$, $j = 1, 2, 3$, $\partial_\nu = \sum_{j=1}^3 \nu_j \partial_j$. Let $\nu_\Gamma(t) = (\nu_{\Gamma,1}(t), \nu_{\Gamma,2}(t), \nu_{\Gamma,3}(t))$, $t \in \Gamma$, be the unit normal vector field on the boundary Γ, which is tangential to the surface $\mathcal{C}$ and directed outside of the surface. We study the following mixed boundary value problem for the Laplace-Beltrami equation

$$\begin{cases} \Delta_{\mathcal{C}} u(t) = f(t), & t \in \mathcal{C}, \\ u^+(\tau) = g(\tau), & \tau \in \Gamma_D, \\ (\partial_{\nu_\Gamma} u)^+(\tau) = h(\tau), & \tau \in \Gamma_N, \qquad \partial_{\nu_\Gamma} := \sum_{j=1}^3 \nu_{\Gamma,j} \mathcal{D}_j. \end{cases} \tag{1}$$

Lax-Milgram Lemma applied to the BVP (1) gives that the BVP (1) has a unique solution in the classical setting $f \in \widetilde{\mathbb{H}}^{-1}(\mathcal{C})$, $g \in \mathbb{H}^{1/2}(\Gamma)$, $h \in \mathbb{H}^{-1/2}(\Gamma)$.

But in some problems, for example in approximation methods, it is important to know the solvability properties in the non-classical setting

$$f \in \widetilde{\mathbb{H}}_p^{s-2}(\mathcal{C}), \quad g \in \mathbb{W}_p^{s-1/p}(\Gamma), \quad h \in \mathbb{W}_p^{s-1-1/p}(\Gamma), \quad 1 < p < \infty, \quad s > \frac{1}{p}. \tag{2}$$

To this end we prove the following.

THEOREM. *Let* $1 < p < \infty$, $s > \dfrac{1}{p}$. *The BVP* (1) *is not Fredholm in the*

non-classical setting (2) *if and only if:*

$$\begin{cases} p \neq \frac{4}{3}, 2, 4 \quad \textit{and} \quad s > \frac{1}{p} \quad \textit{is arbitrary,} \\ p = \frac{4}{3}, \quad s \neq \frac{1}{p} + \frac{1}{4} + k, \quad k = -1, 0, 1, \dots \\ p = 2 \quad \textit{and} \quad s \neq \frac{1}{p} + k, \quad k = 0, 1, \dots, \\ p = 4, \quad s \neq \frac{1}{p} + \frac{3}{4} + k, \quad k = -1, 0, 1, \dots. \end{cases}$$

In particular, the BVP (1) *has a unique solution* u *in the non-classical setting* (2) *if*

$$\frac{4}{3} < p < 4 \quad \textit{and} \quad \frac{1}{p} + \frac{1}{4} < s < \frac{1}{p} + \frac{3}{4}.$$

Multiplier algebras of weighted shifts on directed trees

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Abstract

We study a class of bounded weighted shifts on directed trees with roots focusing on analytic aspects of their theory. We define and study multiplier algebras related to these operators. We show that these algebras consists of coefficients of analytic functions and forms a Banach algebra closed in SOT and WOT topology. From this fact we deduce reflexivity of those weighted shifts on directed trees whose all path-induced spectral-like radii are positive.

This presentation is based on a joint work with P. Budzyński, A. Płaneta and M. Ptak

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High-frequency asymptotic behaviour for the damped wave equation on metric graphs

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Abstract

We consider the damped wave equation on metric graphs and study the localisation of high-frequency spectral abscissas, that is, values around which the real parts of the eigenvalues will cluster, as the corresponding imaginary parts grow to infinity. Under certain conditions on the edge lengths and the coupling conditions, we show that there exists at most a finite number of such spectral abscissas which may be determined by solving a polynomial equation. This is joint work with Jiří Lipovský.

C_0-semigroups of 2-isometries and Dirichlet spaces

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Abstract

The concept of a 2-isometry was introduced by Agler in the early eighties and were characterized in terms of their extension properties. Recall that a bounded linear operator T on a separable, infinite dimensional complex Hilbert space $\mathcal{H}$ is called a *2-isometry* if it satisfies

$$T^{*2}T^2 - 2T^*T + I = 0,$$

where I denotes the identity operator. In addition, such operators are called *analytic* if no nonzero vector is in the range of every power of T. It turns out that M_z, i.e. the multiplication operator by z, acting on the classical Dirichlet space, is a cyclic analytic 2-isometry. But, moreover, Richter proved

that any cyclic analytic 2-isometry is unitarily equivalent to M_z acting on a generalized Dirichlet space $D(\mu)$.

In the context of Richter's theorem, we will establish a similarity between C_0-semigroups of analytic 2-isometries $\{T(t)\}_{t\geq 0}$ acting on a Hilbert space $\mathcal{H}$ and the multiplication operator semigroup $\{M_{\phi_t}\}_{t\geq 0}$ induced by $\phi_t(s) = \exp(-st)$ for s in the right-half plane $\mathbb{C}_+$ acting boundedly on weighted Dirichlet spaces on $\mathbb{C}_+$. As a consequence, we derive a connection with the right shift semigroup $\{S_t\}_{t\geq 0}$

$$S_t f(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq t, \\ f(x-t) & \text{if } x > t, \end{cases}$$

acting on a weighted Lebesgue space on the half line $\mathbb{R}_+$ and will address some applications regarding the study of the invariant subspaces of C_0-semigroups of analytic 2-isometries.

Joint work with Jonathan R. Partington (University of Leeds).

A Hankel matrix acting on spaces of analytic functions

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Abstract

If μ is a positive Borel measure on the interval $[0, 1)$, the Hankel matrix $\mathcal{H}_\mu = (\mu_{n,k})_{n,k\geq 0}$ with entries $\mu_{n,k} = \int_{[0,1)} t^{n+k}\, d\mu(t)$ induces formally the operator

$$\mathcal{H}_\mu(f)(z) = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{\infty} \mu_{n,k} a_k \right) z^n$$

on the space of all analytic functions $f(z) = \sum_{k=0}^{\infty} a_k z^k$, in the unit disc $\mathbb{D}$. This is a natural generalization of the classical Hilbert operator.

In this talk we shall report on several recent works regarding the action of the operator $\mathcal{H}_\mu$ on distinct spaces of analytic functions in $\mathbb{D}$.

Isometries with finite spectrum on continuous functions

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Abstract

Let $\mathcal{X}$ be a complex Banach space and let $T\colon \mathcal{X} \to \mathcal{X}$ be a (necessarily surjective) isometry of period m, that is, $T^m = I$ and $T^k \neq I$ for $k = 1, \ldots, m-1$. The point spectrum of T consists of n distinct mth roots of unity ($n \leq m$), and we can always assume that it contains 1 as an eigenvalue. If $\sigma(T) = \{1, \lambda_1, \ldots, \lambda_{n-1}\}$ and P_i is the projection onto the kernel of $T - \lambda_i I$ then

$$P_0 \oplus P_1 \oplus \cdots \oplus P_{m-1} = I \quad \text{and} \quad P_0 + \lambda_1 P_1 + \cdots + \lambda_{n-1} P_{n-1} = T.$$

In this talk the emphasis will be given to $C_0(\Omega)$, the Banach space of all continuous complex-valued functions on a connected locally compact Hausdorff space Ω vanishing at infinity. By the Banach-Stone theorem, surjective linear isometries on $C_0(\Omega)$ are weighted composition operators. In particular, the cases when the spectrum of a weighted composition operator consists of 2, 3, 4, or 5 elements will be completely described.

This is a joint work with Chih-Neng Liu and Ngai-Ching Wong from the National Sun Yat-sen University, Kaohsiung, Taiwan.

Shape of kernels of block Hankel operators

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Abstract

It is known that for a matrix-valued function $\Phi \in L^2_{M_{n\times m}}$, the kernel of a block Hankel operator H_Φ on a vector-valued Hardy spaceis an invariant subspace for the shift operator.Thus, $\ker H_\Phi = \Theta H^2_{\mathbb{C}^r}$ for a natural number r and an $n \times r$ matrix inner function Θ. It will be shown that the size of the inner function Θassociated with the kernel of block Hankel operator $H_\Phi$is decided bya certainindependency of the columns of the symbol function Φ. As applications, the shape of shift-invariant and backward shift-invariant subspaces of $H^2_{\mathbb{C}^n}$ generated by finite elements will be studied.

One-sided invertibility of infinite band-dominated matrices and their applications

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Abstract

The two-sided and one-sided invertibility of multiplication operators by infinite band-dominated matrices on the spaces $l^p(\mathbb{Z})$ with $p \in [1, \infty]$ is studied. Criteria of two- and one-sided invertibility of such operators are obtained under the condition of slowly oscillating behavior of diagonal entries $a_{n,n+k}$ of matrices as $n \to \pm\infty$, by applying a specific factorization of symbols of these operators at $\pm\infty$. Criteria of the one-sided invertibility of multiplication operators by infinite two-diagonal matrices whose diagonal entries $a_{n,n+k}$ form elements $\{a_{n,n+k}\}_{n\in\mathbb{Z}} \in l^\infty(\mathbb{Z})$ are also established. Applications to studying the two- and one-sided invertibility of Wiener type functional operators on Lebesgue spaces are considered.

Conjugations and reflexivity

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Abstract

Let $\mathcal{H}$ be a complex Hilbert space with an inner product $\langle\cdot,\cdot\rangle$. By F_k we denote the set of all operators of rank at most k. A weak* closed subspace $\mathcal{S} \subset B(\mathcal{H})$ is *k-reflexive* if rank-k operators are linearly dense in $\mathcal{S}_\perp = \{t$–a trace -class operator : $tr(St) = 0$ for all $S \in \mathcal{S}\}$. A subspace $\mathcal{S}$ is called *k-hyperreflexive* if there is a constant $c > 0$ such that

$$\operatorname{dist}(T, \mathcal{S}) \le c \cdot \sup\{|tr(Tt)| : t \in F_k \cap \mathcal{S}_\perp, \|t\|_1 \le 1\},$$

for all $T \in B(\mathcal{H})$. Note that $\operatorname{dist}(T, \mathcal{S})$ is the infimum distance. Recall that C is a *conjugation* on $\mathcal{H}$ if $C : \mathcal{H} \longrightarrow \mathcal{H}$ is an antilinear, isometric involution,

i.e., $\langle Cx, Cy\rangle = \langle y, x\rangle$ for all $x, y \in \mathcal{H}$ and $C^2 = I$. An operator T in $B(\mathcal{H})$ is said to be *C–symmetric* if $CTC = T^*$.

During the talk we will present results concerning reflexivity and hyper-reflexivity of the space of all C–symmetric operators. We will also discuss reflexivity of truncated Toeplitz operators.

The talk is based on joint work with C. Câmara and M. Ptak.

Spectral analysis of the diffusion operator with random jumps from the boundary

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Abstract

Using an operator-theoretic framework in a Hilbert-space setting, we perform a detailed spectral analysis of the one-dimensional Laplacian in a bounded interval, subject to specific non-self-adjoint connected boundary conditions modelling a random jump from the boundary to a point inside the interval. In accordance with previous works, we find that all the eigenvalues are real.As the new results, we derive and analyse the adjoint operator, determine the geometric and algebraic multiplicities of the eigenvalues, write down formulae for the eigenfunctions together with the generalised eigenfunctions and study their basis properties. It turns out that the latter heavily depend on Diophantine properties of the interior point. Finally, we find a closed formula for the metric operator that provides a similarity transform of the problem to a self-adjoint operator.

This is joint work with Martin Kolb.

Remark on operators whose positive parts are similar

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Abstract

Let $\mathcal{H}$ be an infinite dimensional complex Hilbert space and let $\mathcal{L}(\mathcal{H})$ be the algebra of all bounded linear operators. Let $S = U_S|S|$ and $T = U_T|T|$

be the polar decompositions of S and T in $\mathcal{L}(\mathcal{H})$, respectively. We say that S and T have the similar positive parts if $|S|$ and $|T|$ are similar. In this talk, we investigate various properties of S and T whose positive parts are similar. In particular, we provide spectral and local spectral relations of such operators S and T. Finally, we consider some applications and examples for such operators S and T.

Complex symmetric operator matrix and its applications

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Abstract

In this paper, we study conjugation matrices and complex symmetric operator matrices. In particular, we provide conditions for 2×2 operator matrices to be conjugations or complex symmetric on $\mathcal{H} \oplus \mathcal{H}$. Moreover, we give examples of conjugation matrices and complex symmetric operators. Finally, we apply the main results to block Toeplitz operator and completion problems.

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Indefinite generalized numerical ranges: geometric versus algebraic properties

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Abstract

Many problems in quantum physics reduce to the study of expressions of the form $\mathrm{tr}(CU * AU)$, where U is unitary for a fixed pair of operators A and C. The collection of all such complex numbers is known as the C-numerical range of A and has attracted the attention of many authors. A variant of this set, induced by an indefinite selfadjoint operator H, is obtained if U and U* are replaced by a H-unitary operator and its H-adjoint. Some results which illustrate the interplay, in the finite dimensional case, between the geometric properties of this set and the algebraic properties of A, C are presented. Some insight into the C-determinantal range of A induced by H is also given.

This talk is based on a joint work A. Guterman and G. Soares.

Morrey spaces and semigroups of composition operators

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Abstract

For $0 < \lambda < 1$ we define the Morrey spaces $\mathcal{L}^{2,\lambda}$ as

$$\mathcal{L}^{2,\lambda} = \left\{ f \in H^2 \, : \, \sup_{a \in \mathbb{D}} (1-|a|^2)^{1-\lambda} \int_{\mathbb{D}} |f'(z)|^2 (1-|\sigma_a(z)|^2)\, dA(z) < \infty \right\},$$

where σ_a is the Möbius self-map given by $\sigma_a(z) = \frac{a-z}{1-\bar{a}z}$.
Morrey spaces can be seen as a natural extension of $BMOA$.

We study some of the most important properties of these spaces as well as semigroups of composition operators acting on them.

This work is based on a joint work with professors P. Galanopoulos and A. G. Siskakis from Aristotle University of Thessaloniki, Greece.

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Eigenvalues of double cyclical positive matrices

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Abstract

We analyze double cyclical n x n matrices and find the upper bound of the number of eigenvalues in the left (right) half-plane. It solves the question (conjecture) posed by C. Johnson, Z. Price, and I. Spitkovsky.

Maximal success is achieved when all (diagonal) elements are brought to the same position. Roadmap of the proof is an elimination of both left and right extreme positions, bringing them to the center (the geometric mean), and a construction of the thick impermeable wall to keep good root points produced by the diverse multi-set on the right.

This is a joint work with Charles Baker, the Ohio State University; see arXiv 1705.01529.

The Hardy operator and its connection to optimal/sharp embeddings of Sobolev-type spaces into Hölder-type spaces

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Abstract

We discuss different sharp inequalities involving the Hardy operator on the cone of monotone functions. These inequalities will allow us to derive necessary

and sufficient conditions for embeddings of Sobolev-type spaces modelled upon rearrangement invariant Banach function spaces $X(\mathbb{R}^n)$ into generalised Hölder-type spaces. We apply our results to the case when $X(\mathbb{R}^n)$ is the Lorentz-Karamata space $L_{p,q;b}(\mathbb{R}^n)$. In particular, we are able to characterise optimal/sharp embeddings of Bessel potential-type spaces and Sobolev-type spaces into generalized Hölder spaces. As immediate applications of our results we obtain continuity envelopes and give upper estimates for approximation numbers for some related embeddings.

This talk is based on collaboration work with B. Opic (Department of Mathematical Analysis, Charles University) and A. Gogatishvili (Institute of Mathematics, Academy of Sciences of the Czech Republic)

A geometric glimpse of spaces of operators

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Abstract

In this talk, we discuss how the geometry of spaces such as $B(H)$, spin factors or C*-algebras is encoded in their algebraic structure. A particular emphasis will be given to the facial structure of their unit balls. The presentation aims to be self-contained.

This is joint work with C.M. Edwards (The Queen's College, Oxford).

Toeplitz Order

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Abstract

Many problems in the area of the Uncertainty Principle in Harmonic Analysis can be viewed as questions on partial order of the set of inner functions in the upper half-plane. The order is induced by Toeplitz operators and extends the existing natural orderings, such as ordering by division. In my talk I will give basic definitions of Toeplitz order and discuss connections with classical problems, such as the Beurling-Malliavin problem, the Gap and Type problems. At the end I will discuss open questions.

Conjugation in model spaces and asymmetric truncated Toeplitz operators

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Abstract

Let H^2 be the Hardy space on the unit disc, identified as usual with a subspace of L^2 on the unit circle. With any nonconstant inner function θ we associate the model space K^2_θ, defined by $K^2_\theta = H^2 \ominus \theta H^2$. A conjugation is an antilinear, isometric, involution and in K^2_θ there is a natural conjugation $C_\theta \colon K^2_\theta \to K^2_\theta$ given by $C_\theta f(z) = \theta \overline{z f(z)}$.

Let us consider two nonconstant inner functions α and θ such that α divides θ. For a given function $\varphi \in L^2$ we can define an asymmetric truncated Toeplitz operator $A_\varphi \colon K^2_\theta \to K^2_\alpha$ by $A_\varphi f = P_\alpha(\varphi f)$, where $P_\alpha \colon L^2 \to K^2_\alpha$ is the orthogonal projection. The relation between bounded asymmetric truncated Toeplitz operators with L^2 symbols and various conjugations C_θ, C_α, $C_{\alpha, \frac{\theta}{\alpha}}$ will be investigated. The relations lead us to connections with Hankel operators.

Translation-invariant subspaces for weighted L^2 on $\mathbb{R}_+$

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Abstract

In the setting of weighted L^2-spaces, a striking result due to Domar states that the lattice of closed invariant subspaces for $\{S_\tau\}_{\tau \geq 0}$ in $L^2(\mathbb{R}_+, w(t)dt)$ coincides with the lattice of *"standard invariant subspaces"*

$$L^2([a,\infty), w(t)dt) = \{f \in L^2(\mathbb{R}_+, w(t)dt) : f(t) = 0 \text{ a.e } 0 \leq t \leq a\}, \quad (a \geq 0),$$

whenever w satisfies:

1. w positive continuous decreasing in $\mathbb{R}_+$.
2. $\log w$ is concave in $[c, \infty)$, for some $c \geq 0$.

3. $\lim_{t\to\infty} \dfrac{-\log w(t)}{t} = \infty$ and $\lim_{t\to\infty} \dfrac{\log|\log w(t)| - \log t}{\sqrt{\log t}} = \infty$.

In this talk we present an extension of Domar's Theorem to a wider class of weight functions w not fulfilling condition (2), which is replaced by a geometric condition on $\{w(t_n)\}_{n\geq 1}$ for some strictly increasing sequence $\{t_n\}_{n\geq 1} \subset \mathbb{R}_+$ with the uniformly bounded condition $\sup_n(t_{n+1} - t_n) < \infty$. This extension addresses, in some sense, a question posed by Domar.

In addition, we provide an example of a weight function $\tilde{w}$ for which the lattice of closed invariant subspaces for $\{S_\tau\}_{\tau\geq 0}$ in $L^2(\mathbb{R}_+, \tilde{w}(t)dt)$ is non-standard such that $\tilde{w}$ satisfies (3) and the geometric condition is fulfilled only for sequences $\{t_n\}_{\geq 1} \subset \mathbb{R}_+$ with $\sup_n(t_{n+1} - t_n) = \infty$.

This is a joint work with Eva A. Gallardo-Gutiérrez (Universidad Complutense de Madrid, Spain) and Jonathan R. Partington (University of Leeds, U.K.).

Multipliers between sub-Hardy Hilbert spaces

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Abstract

In this joint work with Emmanuel Fricain and Andreas Hartmann, we characterize the multipliers between pairs of sub-Hardy Hilbert spaces of analytic functions on the unit disk. Examples explored are the classical model spaces and the range spaces of co-analytic Toeplitz operators.

Non-accretive Schrödinger operators and exponential decay of their eigenfunctions

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Abstract

We consider non-self-adjoint electromagnetic Schrödinger operators on arbitrary open sets with complex scalar potentials whose real part is not necessarily bounded from below. Under a suitable sufficient condition on the electromagnetic potential, we introduce a Dirichlet realization as a closed densely defined operator with non-empty resolvent set and show that the eigenfunctions corresponding to discrete eigenvalues satisfy an Agmon-type exponential decay.

The talk is based on:

[1] D. Krejčiřík, N. Raymond, J. Royer and P. Siegl: *Non-accretive Schrödinger operators and exponential decay of their eigenfunctions*, Israel Journal of Mathematics, to appear, arXiv:1605.02437.

Wiener-Hopf factorization through an intermediate space and applications

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Abstract

An operator factorization conception is investigated for a general Wiener-Hopf operator $W = P_2 A|_{P_1 X}$ where X, Y are Banach spaces, $P_1 \in \mathcal{L}(X), P_2 \in \mathcal{L}(Y)$ are projectors and $A \in \mathcal{L}(X, Y)$ is invertible. Namely we study a particular factorization of $A = A_- C A_+$ where $A_+ : X \to Z$ and $A_- : Z \to Y$ have certain invariance properties and the cross factor $C : Z \to Z$ splits the "intermediate space" Z into complemented subspaces closely related to the kernel and cokernel of W, such that W is equivalent to a "simpler" operator, $W \sim PC|_{PX}$.

The main result shows equivalence between the generalized invertibility of the Wiener-Hopf operator and this kind of factorization (provided $P_1 \sim P_2$) which implies a formula for a generalized inverse of W. It embraces I.B. Simonenko's generalized factorization of matrix measurable functions in L^p spaces and various other factorization approaches. Various connected theoretical questions are answered such as: How to transform different kinds of factorization into each other? When is W itself the truncation of a cross factor?

As applications we consider interface problems in weak formulation for the n-dimensional Helmholtz equation in $\Omega = \mathbb{R}^n_+ \cup \mathbb{R}^n_-$ (due to $x_n > 0$ or $x_n < 0$, respectively), where the interface $\Gamma = \partial\Omega$ is identified with $\mathbb{R}^{n-1}$ and divided into two parts, Σ and Σ', with different transmission conditions of first and second kind. These two parts are half-spaces of $\mathbb{R}^{n-1}$ (half-planes for $n = 3$). We construct explicitly resolvent operators acting from the interface data into the energy space $H^1(\Omega)$. The approach is based upon the present factorization conception and avoids an interpretation of the factors as unbounded operators. In a natural way, we meet non-isotropic Sobolev spaces which reflect the wedge asymptotic of diffracted waves.

The talk is based upon several papers, particularly on

F.-O. Speck, *Wiener-Hopf factorization through an intermediate space.* Integral Equations Oper. Theory **82** (2015), 395-415.

F.-O. Speck, *A class of interface problems for the Helmholtz equation in* $\mathbb{R}^n$. Math. Meth. Appl. Sciences **40** (2017), 391–403.

On some new results related to and using the "two projections theory"

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Abstract

A canonical representation of a pair of orthogonal projections was obtained by Halmos in his "Two subspaces" paper as early as 1969. I will discuss several recent results which either were obtained by using this representation, or can be sharpened with its help.

A Szegö type theorem for TTOs

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Abstract

Toeplitz operators are compressions of multiplication operators on L^2 spaces to the Hardy space, while Toeplitz matrices compress the multiplications to the space of polynomials. A generalization of these types of operators, called 'truncated Toeplitz' is the compression of multiplications to 'model spaces' (i.e. subspaces of the Hardy space which are invariant under the backward shift) - called *truncated Toeplitz operators*.

The classical theorem of Szego - describes the spectrum of a chain of Toeplitz matrices converging to a Toeplitz operator, relating it to the values of the symbol of the multiplication operator at certain points. In our recent paper with D. TImotin and M. Zarrabi we prove a variant, or generalization of this theorem when this type of 'approximation' is done using truncated Toeplitz operators.

References

[1] D. Sarason, *Algebraic properties of truncated Toeplitz operators*, Oper. Matrices 1 (2007), no. 4, 491–526.

[2] E. Strouse, D. TImotin, M. Zarrabi, *A Szego type theorem for truncated Toeplitz operators*, J. Approx Theory (2017), no.220, 12-30.

Holomorphic Hermite polynomials in two variable - the entangled case

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Abstract

I intend to present the polynomials in question in some detail and the relevant operators of the bipartite quantum oscillator behind.

Beyond truncated Toeplitz operators

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Abstract

Completely nonunitary contractions with defect spaces of dimension 1 have a functional model defined in terms of their characteristic function u, which belongs to the unit ball of H^∞. In case u is inner, the model space becomes $K_u = H^2 \ominus uH^2$, and the model operator S_u is the compression of the shift to K_u. The class $\mathcal{T}_u$ of truncated Toeplitz operators on K_u may be defined by an invariance property with respect to S_u.

We investigate $\mathcal{M}_u$, the class analogous to $\mathcal{T}_u$, in the case of a general (not necessarily inner) function u. It is also characterized by an invariance property with respect to the model operator, and admits an alternate description by means of multiplications with matrix valued functions. Other properties of this class of operators are also investigated.

This is joint work with Hari Bercovici.

Transition asymptotics of Toeplitz determinants

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Abstract

For sufficiently smooth symbols, Szegs theorems describe the asymptotic behavior of Toeplitz determinants. Asymptotic formulas are also known for the determinants of Toeplitz matrices generated by Fisher-Hartwig symbols. Suppose now that the symbol has an extra parameter t with the property that when t goes to zero, the number of Fisher-Hartwig singularities changes. By transition asymptotics (or double scaling limits) of Toeplitz determinants we mean limits of the determinants when the size of the matrices goes to infinity and t goes to zero simultaneously. In this talk, I discuss the known results on transition asymptotics with emphasis on joint work with Kasia Kozlowska, which describes the transition from one Fisher-Hartwig singularity to two Fisher-Hartwig singularities. Motivation for transition asymptotics comes from applications in mathematical physics and random matrix theory. In particular, our results describe a transition in the theory of XY spin chains related to the emptiness formation probability.

Sturm-Liouville problems with transfer condition Herglotz dependent on the eigenparameter

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Abstract

We consider Sturm-Liouville equation $\ell f := -f'' + qf = \lambda f$ on the intervals $(-a, 0)$ and $(0, b)$ with $a, b > 0$ and $q \in L^2(-a, b)$. We impose boundary conditions $f(-a)\cos\alpha = f'(-a)\sin\alpha, f(b)\cos\beta = f'(b)\sin\beta$, where $\alpha \in [0, \pi)$ and

$\beta \in (0, \pi]$, and transmission conditions rationally-dependent on the eigenparameter via $f(0^+) = -(f'(0^+) - f'(0^-)) \sum_{i=1}^{N} \frac{\beta_i^2}{\lambda - \gamma_i}$, $f'(0^-) = (f(0^+) - f(0^-)) \sum_{j=1}^{M} \frac{\alpha_j^2}{\lambda - \delta_j}$, with $\beta_i, \alpha_j > 0$ for $i = 1, \ldots, N$, and $j = 1, \ldots, M$. The geometric multiplicity of the eigenvalues is considered and the cases in which the multiplicity can be 2 are characterized. An example is given to illustrate the cases. A Hilbert space formulation of the above eigenvalue problem as a self-adjoint operator eigenvalue problem in $L^2(-a, b) \bigoplus \mathbb{C}^N \bigoplus \mathbb{C}^M$ is given. A characteristic determinant is defined which has zeros coinciding with the eigenvalues and order of the zeros corresponding to the geometric multiplicity of the eigenvalues. The Green's function is given and related to the resolvent of the Hilbert space operator. Finally spectral asymptotics are given.

Keywords: Sturm-Liouville, Transfer condition.

A variation principle for ground spaces

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Abstract

A variation formula is presented for the ground space projections of a vector space of energy operators in a matrix $*$-algebra. We prove that the ground space projections are the greatest projections of the algebra under certain operator cone constraints. The formula is derived from lattice isomorphisms between normal cones and exposed faces of the state space of the algebra, and between ground space projections.

The vector space of local Hamiltonians is in the focus of quantum many-body physics. The variation formula will be demonstrated with two-local three-bit (commutative) Hamiltonians. A future goal is to understand the lattice of ground spaces of two-local three-qubit (non-commutative) Hamiltonians. Both its combinatorics and topology are unsettled issues.

Reference: arXiv:1704.07675 [math-ph]

Posters

PDLs and Flowcharts in Operator Theory

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Abstract

In recent years, several software applications were made available to the general public with extensive capabilities of symbolic computation. These applications, known as computer algebra systems (CAS), allow to delegate to a computer all, or a significant part, of the symbolic calculations present in many mathematical algorithms. In our work, we use the CAS Mathematica to implement analytical algorithms developed by us and others within the Operator Theory. The main goal of this work is to present some Program Design Languages (PDLs) and flowcharts related with some of those algorithms.

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A singular integral operator with a non-Carleman shift and conjugation

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Abstract

On the Hilbert space $\widetilde{L}_2(\mathbb{T})$ the singular integral operator with non-Carleman shift and conjugation $K = P_+ + (aI + AC)P_-$ is considered, where $P_\pm$ are the Cauchy projectors, $A = \sum\limits_{j=0}^{m} a_j U^j$, a, a_j, $j = \overline{1,m}$, are continuous functions on the unit circle $\mathbb{T}$, U is the shift operator and C is the operator of complex conjugation. An estimate for the dimension of the kernel of the operator K is obtained.

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